EJERCICIOS

A) Resolver las siguientes ecuaciones

1) $2\log_2 x - \log_2(x-16) = 6$
2) $\log x = 1 + \log(22-x)$
3) $2\log(5x+4) - \log 4 = \log(x+4)$
4) $(x^2-5x+9)\log 2 + \log 125 = 3$
5) $\ln(2x-3) + \ln(5-x) = \ln 5$
6) $\ln x = \ln 2 + 2\ln(x-3)$
7) $2\log x = 3 + \log\frac{x}{10}$
8) $\log x + \log(x+3) = 2\log(x+1)$
9) $4\log\left(\frac{x}{5}\right) + \log\left(\frac{625}{4}\right) = 2\log x$

B) Resolver las siguientes ecuaciones

1) $2\log x - 2\log(x+1) = 0$
2) $\log x = \frac{2-\log x}{\log x}$
3) $\log(25-x^3) - 3\log(4-x) = 0$
4) $\frac{\log(16-x^2)}{\log(3x-4)} = 2$
5) $\frac{\log(35-x^3)}{\log(5-x)} = 3$
6) $\log 2 + \log(11-x^2) = 2\log(5-x)$
7) $\log_5 x + \frac{\log_5 125}{\log_5 x} = \frac{7}{2}$
8) $\log_2 x = \log_2 24 - \log_2 12$
9) $\log x + \log(x+15) = 2$

C) Resolver las siguientes ecuaciones

1) $\log\sqrt{3x+4} + \frac{1}{2}\log(5x+1) = 1 + \log 3$
2) $(x^2-4x+7)\log 5 + \log 16 = 4$
3) $3\log x - \log 32 = \log\frac{x}{2}$
4) $2\log x = \log\frac{x}{2} - 1$
5) $5\log\frac{x}{2} + 2\log\frac{x}{3} = 3\log x - \log\frac{32}{9}$
6) $2\log x = 3 + \log\frac{x}{10}$
7) $2\log x - \log(x-16) = 2$
8) $\log\sqrt{3x+1} - \log\sqrt{2x-3} = 1 - \log 5$
9) $\log(5x-3)^2 + \log(2x+3)^2 = 2$

D) Resolver las siguientes ecuaciones

1) $\frac{\log 3 + \log(11-x^3)}{\log(5-x)} = 2$
2) $\log(28-x^3) - 3\log(4-x) = 0$
3) $2\log x - 4\log 2 = \log 32$
4) $5\log x - \log 243 = 4\log\frac{x}{3}$
5) $\log(2x+2) + \log(5x-10) = 2$
6) $\log(3x-2)^{\frac{1}{2}} + \log(x+23)^{\frac{1}{2}} = 1$
7) $\log(x+6) = 1 + \log(x-3)$
8) $\frac{100^{\log x} + 40}{10^{\log x}} = 14$
9) $\frac{\log(152-x^3)}{\log(8-x)} = 3$

E) Hallar el valor de x

1) $\log_x 25 = 2$
2) $\log_x 216 = 3$
3) $\log_x 4 = \frac{1}{2}$
4) $\log_x 4 = -\frac{1}{2}$
5) $\log_x 3 = \frac{1}{2}$
6) $\log_x 343 = 3$
7) $\log_x \frac{1}{64} = -6$
8) $\log_x 5 = -\frac{1}{2}$
9) $\log_x \frac{1}{100} = -2$
10) $\log_x 32 = \frac{5}{2}$
11) $\log_x 81 = -4$
12) $\log_x 49 = 2$

F) Calcular

1) $\log_4 7$
2) $\log_5 12$
3) $\log_3 16$
4) $\log_6 13$
5) $\log_{\frac{1}{2}} 15$

G) Calcular en función de $\log 2$ y $\log 3$

1) $\log 2{,}88$
2) $\log\sqrt{5{,}76}$
3) $\log\frac{10{,}8}{\sqrt{14{,}4}}$
4) $\log\left(\sqrt{3{,}2}\cdot\sqrt{1{,}6}\right)$
5) $\log\frac{1}{6561}$
6) $\log\sqrt[4]{781{,}25}$
7) $\log\frac{3{,}2^3\cdot 0{,}64^5}{0{,}0125\cdot\sqrt[4]{80^3}}$
8) $\log\sqrt[3]{\frac{9}{5}}$
9) $\log\frac{1{,}25}{\sqrt{0{,}32}}$

H) Calcular aplicando las propiedades

1) $\log_2\frac{\sqrt[6]{64}\cdot 4^2}{2^5\cdot\sqrt[3]{512}}$
2) $\log_3\frac{27\cdot\sqrt{729}}{81\cdot\sqrt[3]{27}}$
3) $\log_5\frac{25\cdot\sqrt[4]{625}}{125}$
4) $\log_7\frac{49\cdot\sqrt[3]{343}}{\sqrt{2401}}$

A/

1) $2\log_2 x - \log_2(x-16) = 6$

$\log_2 \frac{x^2}{x-16} = \log_2 64$

$\frac{x^2}{x-16} = 64$

$x^2 = 64x - 1024$

$x^2 - 64x + 1024 = 0$

$x = \frac{64 \pm \sqrt{4096-4096}}{2}$

$\boxed{x = 32}$

2) $\log x = 1 + \log(22-x)$

$\log x = \log[10\cdot(22-x)]$

$x = 220 - 10x$

$11x = 220$

$x = \frac{220}{11} = 20$

$\boxed{x = 20}$

3) $2\log(5x+4) - \log 4 = \log(x+4)$

$\log(5x+4)^2 - \log 4 = \log(x+4)$

$\log \frac{(5x+4)^2}{4} = \log(x+4)$

$(5x+4)^2 = 4(x+4)$

$25x^2 + \cancel{16} + 40x = 4x + \cancel{16}$

$25x^2 + 36x = 0$

$x(25x+36) = 0$

$\boxed{x=0}$; $25x+36=0$

Sí $\quad x = -\frac{36}{25}$ NO

4) $(x^2-5x+9)\log 2 + \log 125 = 3$

$\log 2^{x^2-5x+9} = \log 1000 - \log 125$

$\log 2^{x^2-5x+9} = \log \frac{1000}{125}$

$2^{x^2-5x+9} = 2^3$

$x^2-5x+9 = 3$

$x^2-5x+6=0$

$\boxed{x=2 \quad x=3}$

5) $\ln(2x-3) + \ln(5-x) = \ln 5$

$\ln[(2x-3)(5-x)] = \ln 5$

$(2x-3)(5-x) = 5$

$10x - 2x^2 - 15 + 3x = 5$

$-2x^2 + 13x - 20 = 0$

$2x^2 - 13x + 20 = 0$

$x = \frac{13 \pm \sqrt{169-160}}{4}$

$x = \frac{13 \pm \sqrt{9}}{4} = \begin{cases} \frac{13+3}{4} = \boxed{4} \\ \frac{13-3}{4} = \boxed{\frac{5}{2}} \end{cases}$

6) $\ln x = \ln 2 + 2\ln(x-3)$

$\ln x = \ln[2\cdot(x-3)^2]$

$x = 2(x^2+9-6x)$

$x = 2x^2 + 18 - 12x$

$2x^2 - 13x + 18 = 0$

$x = \frac{13 \pm \sqrt{169-144}}{4}$

$x = \frac{13 \pm \sqrt{25}}{4} = \begin{cases} \frac{13+5}{4} = \boxed{\frac{9}{2}} \\ \frac{13-5}{4} = 2 \text{ NO} \end{cases}$

7) $2\log x = 3 + \log\frac{x}{10}$

$\log x^2 = \log(100x)$

$x^2 = 100x$

$x^2 - 100x = 0$

$x(x-100) = 0$

$x = 0$; $\boxed{x=100}$

NO

8) $\log x + \log(x+3) = 2\log(x+1)$

$\log[x\cdot(x+3)] = \log(x+1)^2$

$x(x+3) = x^2 + 1 + 2x$

$\cancel{x^2} + 3x = \cancel{x^2} + 1 + 2x$

$\boxed{x=1}$

9) $4\log\frac{x}{5} + \log\frac{625}{4} = 2\log x$

$\log\left[\left(\frac{x}{5}\right)^4 \cdot \frac{625}{4}\right] = \log x^2$

$\frac{x^4}{4} = x^2$; $4x^2 = x^4$; $4x^2 - x^4 = 0$

$x^2(4-x^2) = 0$

$x = 0$; $4 - x^2 = 0$ $\quad \boxed{x=+2}$

NO $\quad -x^2 = -4$ $\quad x = -2$ NO

B/

1) $2\log x - 2\log(x+1) = 0$

$\log x^2 = \log(x+1)^2$

$x^2 = (x+1)^2$

$\cancel{x^2} = \cancel{x^2} + 1 + 2x$

$x = -\frac{1}{2}$ No tiene soluc.

3) $\log(25-x^3) - 3\log(4-x) = 0$

$\log \frac{25-x^3}{(4-x)^3} = \log 1$

$25 - \cancel{x^3} = 64 - 48x + 12x^2 - \cancel{x^3}$

$-12x^2 + 48x - 39 = 0$

$12x^2 - 48x + 39 = 0$

$x = 2 \pm \frac{\sqrt{3}}{2}$

4) $\frac{\log(16-x^2)}{\log(3x-4)} = 2$

$\log(16-x^2) = \log(3x-4)^2$

$\cancel{16} - x^2 = 9x^2 + \cancel{16} - 24x$

$-10x^2 + 24x = 0$

$5x^2 - 12x = 0$

$x(5x-12) = 0$

$x = 0$ NO ; $\boxed{x = \frac{12}{5}}$

6) $\log 2 + \log(11-x^2) = 2\log(5-x)$

$\log[2\cdot(11-x^2)] = \log(5-x)^2$

$22 - 2x^2 = 25 + x^2 - 10x$

$-3x^2 + 10x - 3 = 0$

$3x^2 - 10x + 3 = 0$

$x = \frac{10 \pm \sqrt{100-36}}{6}$

$x = \frac{10 \pm \sqrt{64}}{6} = \begin{cases} \frac{10+8}{6} = \boxed{3} \\ \frac{10-8}{6} = \boxed{\frac{1}{3}} \end{cases}$

8) $\log_2 x = \log_2 24 - \log_2 12$

$\log_2 x = \log_2 \frac{24}{12}$

$\boxed{x = 2}$

2) $\log x = \frac{2 - \log x}{\log x}$

Cambio $\log x = t$

$t = \frac{2-t}{t}$; $t^2 = 2 - t$

$t^2 + t - 2 = 0$

$t = \frac{-1 \pm \sqrt{1+8}}{2} = \begin{cases} \frac{-1+3}{2} = 1 \\ \frac{-1-3}{2} = -2 \end{cases}$

$\log x = 1$; $\log x = -2$

$10^1 = x$; $10^{-2} = x$

$\boxed{x = 10}$; $\boxed{x = \frac{1}{100}}$

5) $\frac{\log(35-x^3)}{\log(5-x)} = 3$

$\log(35-x^3) = \log(5-x)^3$

$35 - \cancel{x^3} = 125 - 75x + 15x^2 - \cancel{x^3}$

$-15x^2 + 75x - 90 = 0$

$x^2 - 5x + 6 = 0$

$\boxed{x = 2}$; $\boxed{x = 3}$

7) $\log_5 x + \frac{\log_5 125}{\log_5 x} = \frac{7}{2}$

Cambio $\log_5 x = t$

$t + \frac{3}{t} = \frac{7}{2}$; $2t^2 + 6 = 7t$

$2t^2 - 7t + 6 = 0$

$t = \frac{7 \pm \sqrt{49-48}}{4}$

$t = \frac{7 \pm \sqrt{1}}{4} = \begin{cases} \frac{7+1}{4} = 2 \\ \frac{7-1}{4} = \frac{3}{2} \end{cases}$

$\log_5 x = 2$

$5^2 = x \Rightarrow \boxed{x = 25}$

$\log_5 x = \frac{3}{2}$

$x = \sqrt{125} = \boxed{5\sqrt{5}}$

9) $\log x + \log(x+15) = 2$

$\log[x\cdot(x+15)] = \log 100$

$x^2 + 15x = 100$

$x^2 + 15x - 100 = 0$

$x = \frac{-15 \pm \sqrt{225+400}}{2}$

$x = \frac{-15 \pm \sqrt{625}}{2} = \begin{cases} \frac{-15+25}{2} = \boxed{5} \\ \frac{-15-25}{2} = -20 \text{ No} \end{cases}$

C/

① $\log\sqrt{3x+4} + \frac{1}{2}\log(5x+1) = 1 + \log 3$

$\log\left[\sqrt{3x+4}\cdot\sqrt{5x+1}\right] = \log(10\cdot 3)$

$\sqrt{15x^2+23x+4} = 30$

$15x^2+23x+4 = 900$

$15x^2+23x-896 = 0$

$x = \frac{-23\pm\sqrt{529+53760}}{30}$

$x = \frac{-23\pm\sqrt{54289}}{30} = \begin{cases} \frac{-23+233}{30} = \boxed{7 \text{ Si}} \\ \frac{-23-233}{30} = -\frac{128}{15} \text{ NO} \end{cases}$

② $(x^2-4x+7)\log 5 + \log 16 = 4$

$\log 5^{x^2-4x+7} = \log\frac{10.000}{16}$

$5^{x^2-4x+7} = 625$

$x^2-4x+7 = 4$

$x^2-4x+3 = 0$

$x = \frac{4\pm\sqrt{16-12}}{2} = \begin{cases} \frac{4+2}{2} = 3 \\ \frac{4-2}{2} = 1 \end{cases}$

③ $3\log x - \log 32 = \log\frac{x}{2}$

$\log\frac{x^3}{32} = \log\frac{x}{2}$

$\frac{x^3}{32} = \frac{x}{2} \rightarrow 2x^3 = 32x$

$x^3-16x = 0$

$x(x^2-16) = 0$

$\boxed{x=0}$ NO ; $x^2-16=0$

$\boxed{x=+4}$ Si ; $\boxed{x=-4}$ NO

④ $2\log x = \log\frac{x}{2} - 1$

$\log x^2 = \log\left(\frac{x}{2} : 10\right)$

$x^2 = \frac{x}{20} \rightarrow 20x^2 = x$

$20x^2 - x = 0$

$x(20x-1) = 0$

$\boxed{x=0}$ NO ; $20x-1=0$

$\boxed{x=\frac{1}{20}}$ Si

⑤ $5\log\frac{x}{2} + 2\log\frac{x}{3} = 3\log x - \log\frac{32}{9}$

$\log\left(\frac{x^5}{32}\cdot\frac{x^2}{9}\right) = \log\left(x^3 : \frac{32}{9}\right)$

$\frac{x^7}{9\cdot 32} = \frac{9x^3}{32} \Rightarrow 32x^7 = 81\cdot 32x^3$

$32x^7 - 81\cdot 32x^3 = 0$

$32x^3(x^4-81) = 0$

$\boxed{x=0}$ NO ; $\boxed{x=+3}$ Si ; $\boxed{x=-3}$ NO

⑥ $2\log x = 3 + \log\frac{x}{10}$

$\log x^2 = \log\left(1000\cdot\frac{x}{10}\right)$

$x^2 = 100x$

$x^2 - 100x = 0$

$x(x-100) = 0$

$\boxed{x=0}$ NO ; $\boxed{x=100}$ Si

⑦ $2\log x - \log(x-16) = 2$

$\log\frac{x^2}{x-16} = \log 100$

$\frac{x^2}{x-16} = 100$

$x^2 = 100(x-16)$

$x^2-100x+1600 = 0$

$x = \frac{100\pm\sqrt{10000-6400}}{2}$

$x = \frac{100\pm\sqrt{3600}}{2} = \begin{cases} \frac{100+60}{2} = \boxed{80} \\ \frac{100-60}{2} = \boxed{20} \end{cases}$

⑧ $\log\sqrt{3x+1} - \log\sqrt{2x-3} = 1 - \log 5$

$\log\frac{\sqrt{3x+1}}{\sqrt{2x-3}} = \log\frac{10}{5}$

$\sqrt{\frac{3x+1}{2x-3}} = 2$

$\frac{3x+1}{2x-3} = 4$

$3x+1 = 8x-12$

$-5x = -13$

$x = \boxed{\frac{13}{5}}$

⑨ $\log(5x-3)^2 + \log(2x+3)^2 = 2$

$\log\left[(5x-3)^2\cdot(2x+3)^2\right] = \log 100$

$100x^4+180x^3-99x^2-162x+81 = 100$

ó también

$\left[(5x-3)(2x+3)\right]^2 = 100$

$(5x-3)(2x+3) = \pm 10$

$10x^2+9x-19 = 0$

$x = 1$; $x = -\frac{19}{10}$; $x = \frac{\sqrt{41}-9}{20}$

$x = \frac{-9-\sqrt{41}}{20}$

D/

① $\dfrac{\log 3+\log(11-x^3)}{\log(5-x)}=2$

$\log[3\cdot(11-x^3)]=\log(5-x)^2$

$33-3x^3=25+x^2-10x$

$-3x^3-x^2+10x+8=0$

	-3	-1	10	8
2		-6	-14	-8
	-3	-7	-4	$\boxed{0}$
-1		3	4	
	-3	-4	$\boxed{0}$	

$(x-2)(x+1)(-3x-4)=0$

$\boxed{x=2}$; $\boxed{x=-1}$; $\boxed{x=-\frac{4}{3}}$

Si Si Si

② $\log(28-x^3)-3\log(4-x)=0$

$\log(28-x^3)=\log(4-x)^3$

$28-\cancel{x^3}=64-48x+12x^2-\cancel{x^3}$

$-12x^2+48x-36=0$

$-x^2+4x-3=0$

$x^2-4x+3=0$

$x=\dfrac{4\pm\sqrt{16-12}}{2}=\dfrac{4\pm\sqrt{4}}{2}=\begin{cases}\frac{4+2}{2}=\boxed{3} & \text{Si}\\ \frac{4-2}{2}=\boxed{1} & \text{Si}\end{cases}$

③ $2\log x-4\log 2=\log 32$

$\log\dfrac{x^2}{16}=\log 32$

$\dfrac{x^2}{16}=32$

$x^2=2^9$

$\boxed{x=+16\sqrt{2}}$ Si

$x=-16\sqrt{2}$ No

④ $5\log x-\log 243=4\log\dfrac{x}{3}$

$\log\dfrac{x^5}{243}=\log\dfrac{x^4}{3^4}$

$\dfrac{x^5}{3^5}=\dfrac{x^4}{3^4}\rightarrow\dfrac{x^5}{x^4}=\dfrac{3^5}{3^4}$

$\boxed{x=3}$ Si

⑤ $\log(2x+2)+\log(5x-10)=2$

$\log[(2x+2)\cdot(5x-10)]=\log 10^2$

$10x^2-10x-20=100$

$10x^2-10x-120=0$

$x^2-x-12=0$

$x=\dfrac{1\pm\sqrt{1+48}}{2}=\dfrac{1\pm\sqrt{49}}{2}=\begin{cases}\frac{1+7}{2}=\boxed{4} & \text{Si}\\ \frac{1-7}{2}=\boxed{-3} & \text{No}\end{cases}$

⑥ $\log\sqrt{3x-2}+\log\sqrt{x+23}=1$

$\log\sqrt{(3x-2)(x+23)}=\log 10$

$\left(\sqrt{3x^2-67x-46}\right)^2=(10)^2$

$3x^2+67x-46=100$

$3x^2+67x-146=0$

$x=\dfrac{-67\pm\sqrt{4489+1752}}{6}=\dfrac{67\pm\sqrt{6241}}{6}$

$x=\begin{cases}\frac{-67+79}{6}=\boxed{2} & \text{Si}\\ \frac{-67-79}{6}=\boxed{-\frac{73}{3}} & \text{No}\end{cases}$

⑦ $\log(x+6)=1+\log(x-3)$

$\log(x+6)=\log 10+\log(x-3)$

$\log(x+6)=\log[10\cdot(x-3)]$

$x+6=10x-30$

$-9x=-36$

$\boxed{x=4}$ Si

⑧ $\dfrac{100^{\log x}+40}{10^{\log x}}=14$; $\dfrac{10^{\log x^2}+40}{10^{\log x}}=14$

$\dfrac{x^2+40}{x}=14\rightarrow x^2+40=14x$

$x^2-14x+40=0$

$x=\dfrac{14\pm\sqrt{196-160}}{2}=\dfrac{14\pm\sqrt{36}}{2}=\begin{cases}\frac{14+6}{2}=\boxed{10} & \text{Si}\\ \frac{8}{2}=\boxed{4} & \text{Si}\end{cases}$

⑨ $\dfrac{\log(152-x^3)}{\log(8-x)}=3$

$\log(152-x^3)=\log(8-x)^3$

$152-\cancel{x^3}=512-192x+24x^2-\cancel{x^3}$

$-24x^2+192x-360=0$

$x^2-8x+15=0$

$x=\dfrac{8\pm\sqrt{64-60}}{2}=\dfrac{8\pm\sqrt{4}}{2}=\begin{cases}\frac{8+2}{2}=\boxed{5} & \text{Si}\\ \frac{8-2}{2}=\boxed{3} & \text{Si}\end{cases}$